Announcements

1) HW 2 – webwork due tomorrow

2) Quiz on Thursday, over 13.1 & 13.2

3) Don't use "e-mail instructor" button on Webwork, e-mail me directly.

Normal Vectors and Equations

In the equation

$$\langle A, B, C \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

determining a plane in $\mathbb{R}^3$, the vector $\langle A, B, C \rangle$ is called the normal vector.

If the equation for the plane is given by

$$Ax + By + Cz + D = 0,$$

then $\langle A, B, C \rangle$ is the normal vector.

Note: Any scalar multiple of a normal vector is also a normal vector (except $\langle 0,0,0\rangle$!)

Given $Ax+By+Cz+D=0$, if $C\neq 0$, then this is the graph of a function

$$\boxed{z=\frac{-D-Ax-By}{C}}$$

Example 1: Given the plane

$$3x - 8y + 13z - 2 = 0,$$

find a normal vector whose x-coordinate is 6 and express the plane as a function $z = f(x,y)$.

One normal vector: $\langle 3, -8, 13 \rangle$

Multiply by 2: $\boxed{\langle 6, -16, 26 \rangle}$

$$3x - 8y + 13z - 2 = 0$$

yields

$$\boxed{z = \frac{2 - 3x + 8y}{13}}$$

Parallel and Intersecting Planes

Two planes are parallel if their normal vectors are parallel and their constant coefficients are different.

i.e $z = 0$ and $z = 2$ are parallel

(normal vectors $\langle 0, 0, 1 \rangle$)

Just like lines in 2-D, two planes in 3-D are either parallel or intersecting: if they intersect, they do so in a line.

Example 2: Find out whether the planes

$$3x - 4y - z + 3 = 0$$

and

$$-8x + y + 13 = 0$$

are parallel or intersecting; if they intersect, find an equation (in vector form) for the line of intersection.

Normal vector for first plane

$= \langle 3, -4, -1 \rangle$

Normal vector for second plane

$= \langle -8, 1, 0 \rangle$

These are not parallel vectors, so the planes are not parallel. So the planes intersect.

To find the line of intersection, set the equations equal to eachother:

$$3x - 4y - z + 3 = 0$$

$$= -8x + y + 13$$

Solve for z:

$$z = 11x - 5y - 10$$

Plug into first equation.

$$3x - 4y - (11x - 5y - 10) + 3$$

$$= 0$$

$$-8x + y + 13 = 0.$$

Unhelpful!

Use this equation (is the equation for the second plane) to solve for y:

$$y = 8x - 13.$$

Plug into first equation:

$$3x - 4(8x-13) - z + 3 = 0,$$

so

$$z = -29x + 55$$

We now know:

$$y = 8x - 13$$

$$z = -29x + 55$$

Substituting t for x_1 I claim that $\langle t, 8t-13, -29t+55\rangle$ is the equation for the line.

Check by plugging into both equations. It will work!

vector equation:

$$\boxed{t\langle 1, 8, -29\rangle + \langle 0, -13, 55\rangle}$$

Thought Experiment

Normal vectors for our two planes were

$$\langle 3, -4, -1 \rangle$$

and $\langle -8, 1, 0 \rangle$.

Take their cross product.

Depending on which goes first, you get

$$\langle 1, 8, -29 \rangle$$

or $\langle -1, -8, 29 \rangle$

This is the direction vector for our line of intersection! You can always find the direction vector by taking the cross product of the normal vectors of the planes, since the normal vectors are orthogonal to any vector on the translates of either plane to the origin.

To get the equation of the line, you just need to find a point on both planes.

Pick a z value, the solve for x and y-values in the equations for the planc.

Distance from a point to a plane

Suppose a plane has equation

$Ax+By+Cz+D=0$ and

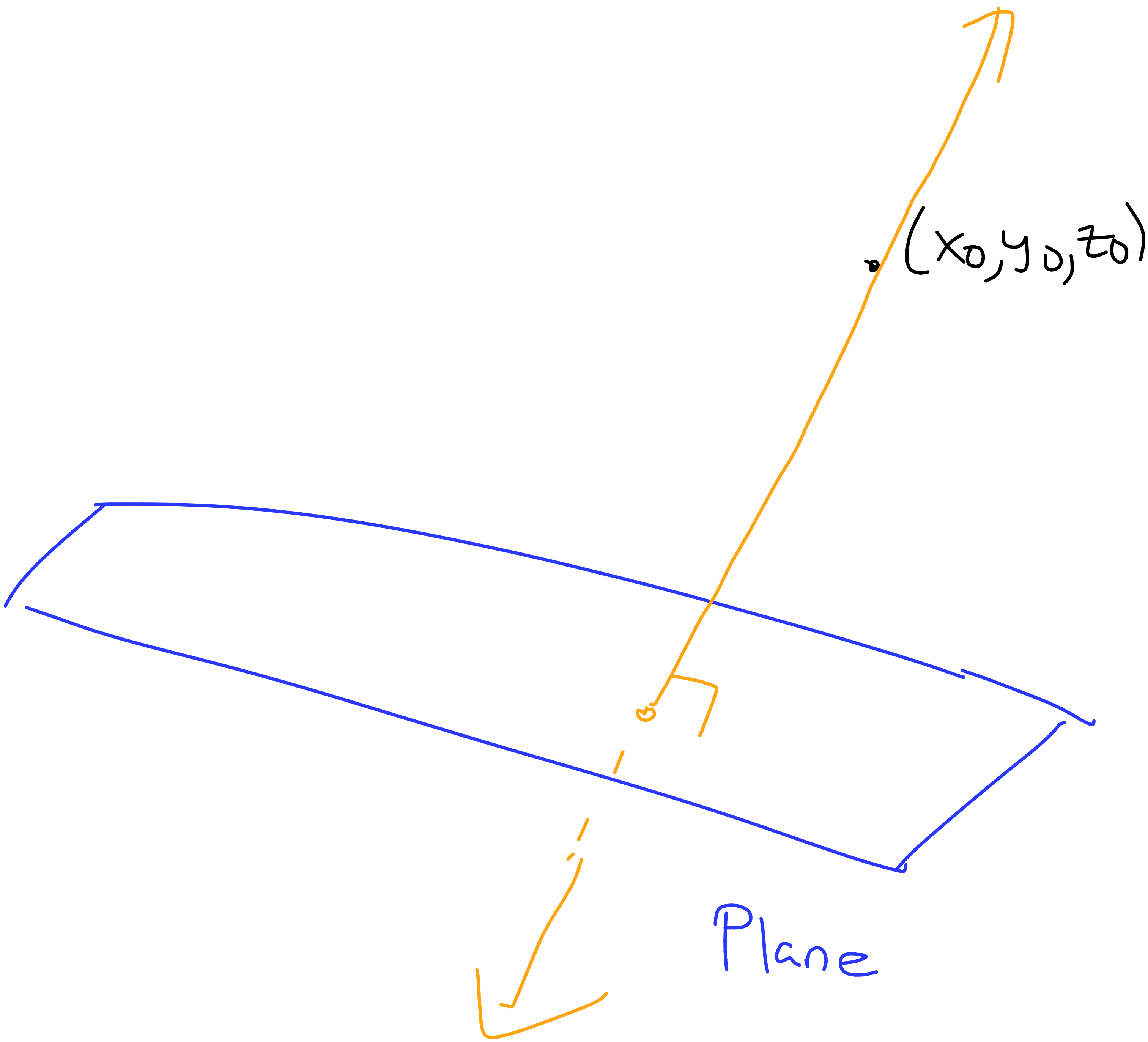
$(x_0, y_0, z_0) \in \mathbb{R}^3$. The

distance from (x_0, y_0, z_0) to

the plane is given by

$$\boxed{d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2+B^2+C^2}}}$$

Q: How would you get this formula?



Find the line orthogonal to the plane that passes through (x_0, y_0, z_0). You can take the direction vector to be the normal vector for the plane. The equation for the line is then

$$\boxed{t\langle A, B, C\rangle + \langle x_0, y_0, z_0\rangle}$$

Write as

$$\langle \underbrace{tA+x_0}_{x}, \underbrace{tB+y_0}_{y}, \underbrace{tC+z_0}_{z} \rangle$$

Plug (x,y,z) back into the equation for the plane to find the point of intersection between the line and the plane.

Then use the distance formula for points to get the equation.

Distance Formula:

If (x_0, y_0, z_0) and (x_1, y_1, z_1) are points in $\mathbb{R}^3$, the (Euclidean) distance between them is

$$\boxed{\sqrt{(x_0-x_1)^2+(y_0-y_1)^2+(z_0-z_1)^2}}$$

Example 3: Find the distance between $(1,-1,4)$ and the plane given by

$$2x - 9y + 10z - 1 = 0$$

Use formula:

$$d = \frac{|1\cdot 2 + (-1)\cdot(-9) + 4\cdot(10) - 1|}{\sqrt{1^2 + (-1)^2 + 4^2}}$$

$$= \boxed{\frac{50}{\sqrt{18}}}$$

Cylinders and Quadric Surfaces

(Section 12.6)

In $\mathbb{R}^2$, $x^2+y^2=1$ is the graph of a circle.

(radius 1, center $(0,0)$)

What about in $\mathbb{R}^3$?

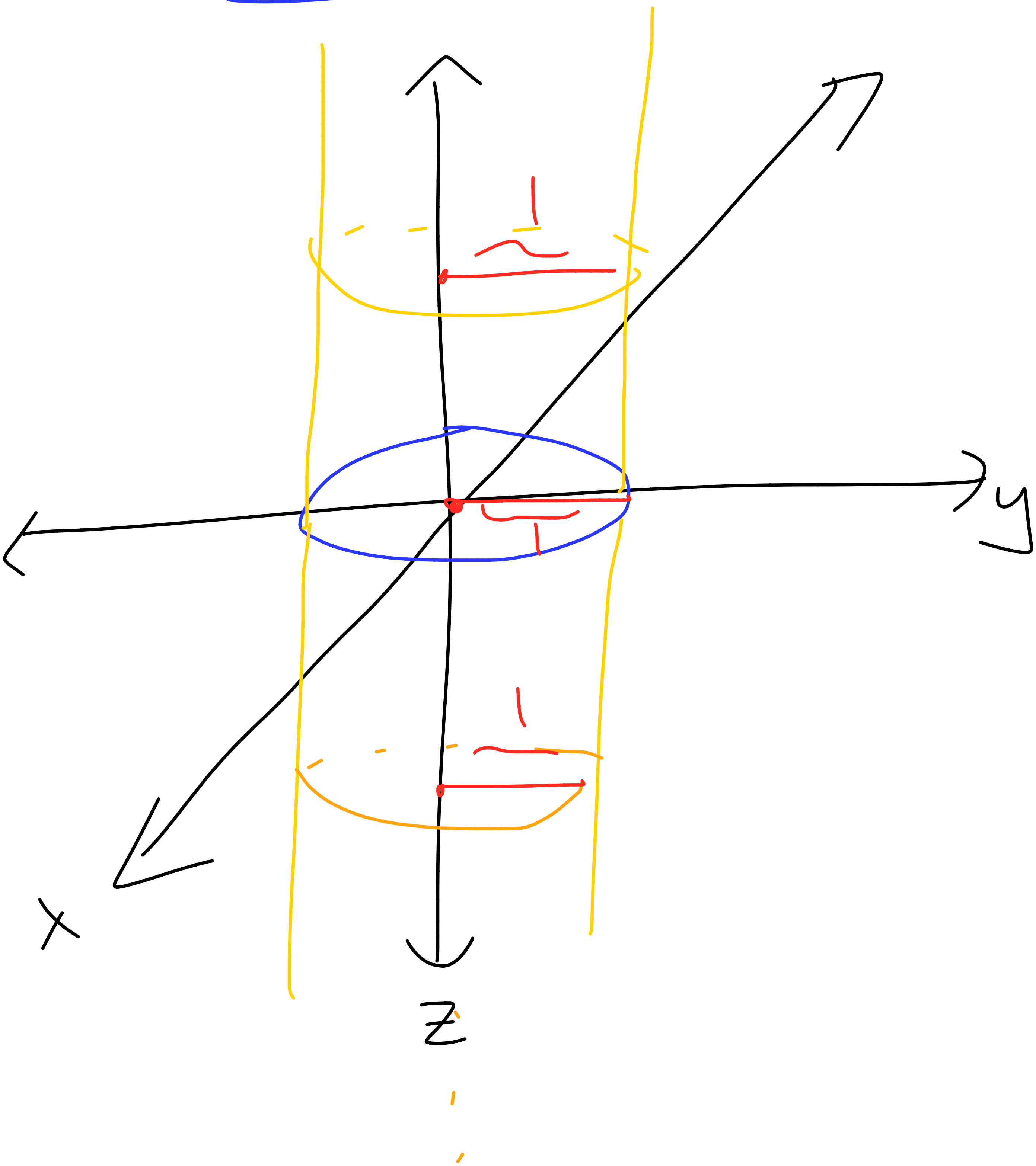
$x^2+y^2=1$ gives a cylinder!

Why? Graph is all points (x,y,z) that satisfy $x^2+y^2=1$.

z is irrelevant to the equation, so we can choose it to be anything!

Picture

$x^2+y^2=1$



Quadric Surfaces

The graph of a second degree equation in x, y, and z

Example 4: $z = x^2 + y^2$

Pick $z = 2$

$2 = x^2 + y^2$ (circle)

You get circles of ever-changing radii except for $z = 0$ (point) and negative values (no graph)

This is the equation of a what?

If $x = 0$, then

$z = y^2$ (parabola)

If $y = 1$,

$z = x^2 + 1$ (parabola)

This is called a paraboloid!

Example 5 : $z^2 = x^2 + y^2$

For all choices of z

except $z=0$ (point),

we get a circle.

$$z = 1$$

$$1 = x^2 + y^2$$

If $x = 0$, we get

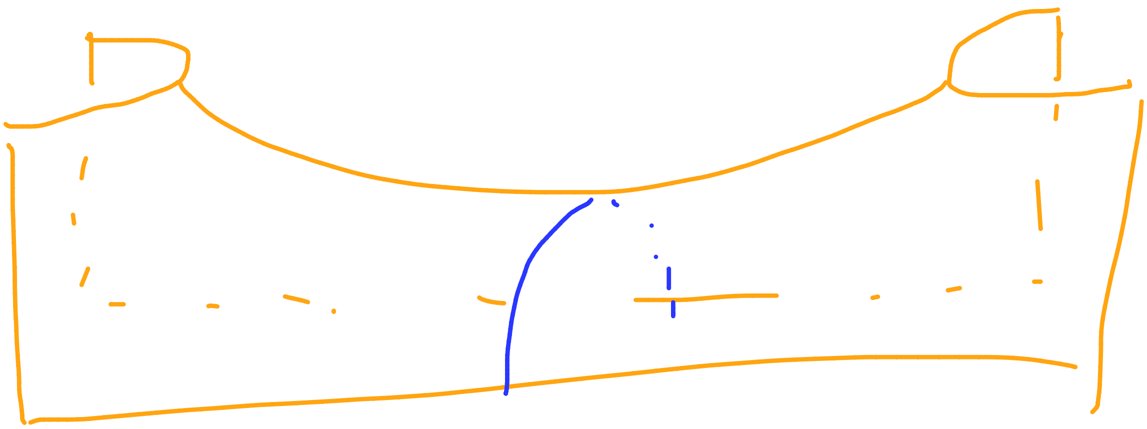
$$\underbrace{z^2 = y^2}$$

either $z = y$ or $z = -y$

two lines

The graph is two Cones (not the graph of a function)

Example 6 : $z = x^2 - y^2$



hyperbolic paraboloid